bl2086 的問題

About **PDE**:
Walter A. Strauss, Partial Differential Equations, An Introduction, 2nd Edition P.41
Show that the wave equation ($u_{tt} = c^2 u_{xx}$) has the following invariance properties.
(a) Any translate $u(x - y,\ t)$, where y is fixed, is also a solution.
SOLUTION:
Let $v(x,\ t) = u(x - y_0,\ t)$, then $v_t = u_t(x - y_0,\ t)$, and $v_{tt} = u_{tt}(x - y_0,\ t) = c^2 u_{xx}(x - y_0,\ t)$
$v_x = u_x(x - y_0,\ t)$, $v_{xx} = u_{xx}(x - y_0,\ t)$,
So $v_{tt} = u_{tt}(x - y_0,\ t) = c^2(u_{xx}(x - y_0,\ t)) = c^2 v_{xx}$ Q.E.D.
(b) Any derivative, say u_x, of a solution is also a solution.
SOLUTION:
For $u_{tt} = c^2 u_{xx}$, let $v = u_x$, then $v_{tt} = u_{xtt} = (u_{tt})_x = (c^2 u_{xx})_x = c^2 u_{xxx} = c^2(u_x)_{xx} = c^2 v_{xx}$ Q.E.D.
(c) The dilated function $u(ax,\ at)$ is also a solution, for any constant a.
SOLUTION:
For $u_{tt} = c^2 u_{xx}$, Let $v(x,\ t) = u(ax,\ at)$, then $v_t = au_t(ax,\ at)$, $v_{tt} = (au_t(ax,\ at))_t = a^2 u_{tt}(ax,\ at) = a^2 c^2 u_{xx}(ax,\ at)$,
$v_x = (u_(ax,\ at))_x = au_x(ax,\ at)$, $v_{xx} = (au_x(ax,\ at))_x = a^2 u_{xx}(ax,\ at)$
So $v_{tt} = a^2 c^2 u_{xx}(ax,\ at) = c^2(a^2 u_{xx}(ax,\ at)) = c^2 v_{xx}$ Q.E.D.